Electromagnetic

Lecture (1)

"Introduction"

2nd year

communications

Lecture (1)
Introduction

⇛ Contents :

Part (1) : Coordinate systems.

Part (2) : Important definetions.

Part (1)
Coordinate systems

Cart. | Cyl. | Sph.

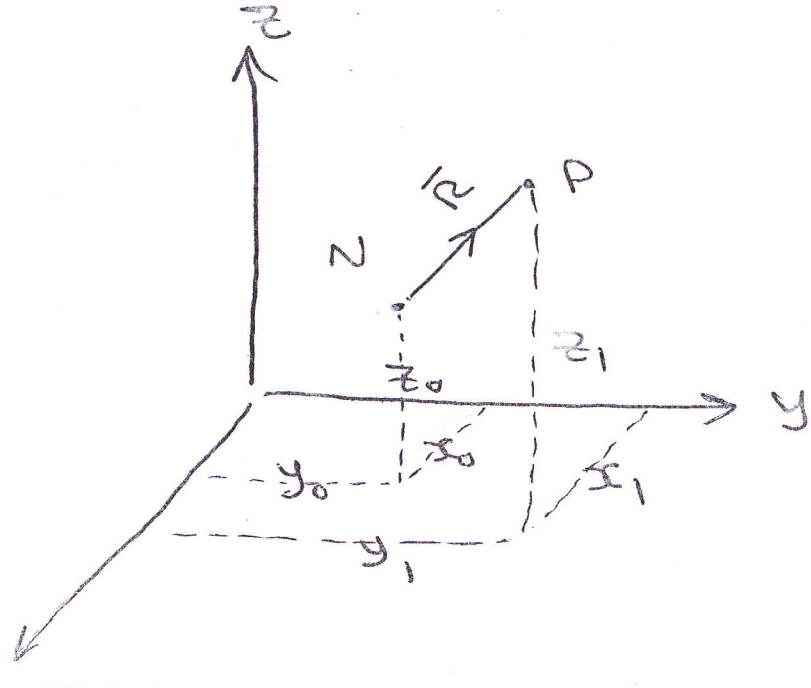
Coordinate systems

(1) Cartezian Coordinate sys :

Points :

P (x_1, y_1, z_1)

N (x_0, y_0, z_0)



- Vector $\bar{R}$ from N to P :

$$\bar{R} = P - N = (x_1, y_1, z_1) - (x_0, y_0, z_0)$$

$$= (x_1 - x_0)\hat{x} + (y_1 - y_0)\hat{y} + (z_1 - z_0)\hat{z}$$

- The mag. of $\bar{R}$:

$$|R| = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$= \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$$

- The unit vector of $\bar{R}$:

$$\hat{R} = \frac{\bar{R}}{|R|} = \frac{(x_1 - x_0)\hat{x} + (y_1 - y_0)\hat{y} + (z_1 - z_0)\hat{z}}{\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}}$$

ex. 1 :

Point A = (0, 3, 2) and B = (3, -3, -2)

Find : (a) $\bar{R}$ from A to B.

(b) represent $\bar{R}$ by its mag. and unit vector.

Solution

(a) $\bar{R} = B - A = 3\hat{x} - 6\hat{y} - 4\hat{z}$

(b) $\Rightarrow \boxed{\bar{R} = |R|\,\hat{R}} \Leftarrow$

$$|R| = \sqrt{9+36+16} = \sqrt{61}$$

$$\hat{R} = \frac{\bar{R}}{|R|} = \frac{3}{\sqrt{61}}\hat{x} - \frac{6}{\sqrt{61}}\hat{y} - \frac{4}{\sqrt{61}}\hat{z}$$

$$\therefore \bar{R} = \sqrt{61}\left[\frac{3}{\sqrt{61}}\hat{x} - \frac{6}{\sqrt{61}}\hat{y} - \frac{4}{\sqrt{61}}\hat{z}\right]$$

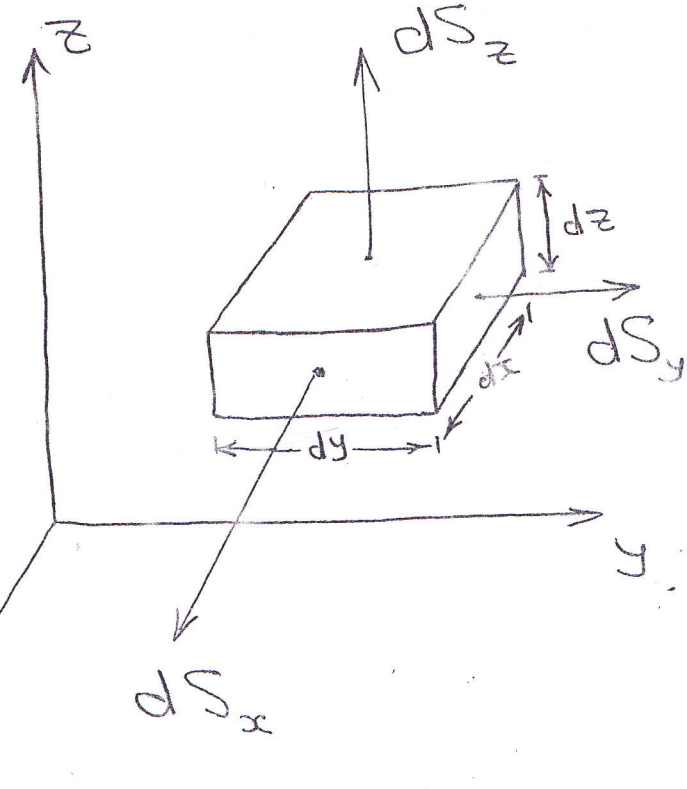
(b) Volume and surface area elements:

$dS_x = dy\,dz$

$dS_y = dx\,dz$

$dS_z = dx\,dy$

$dV = dx\,dy\,dz$



S: Surface area

S_x: Surface area $\perp x$.

- dS_x : Small element of Surface S_x.
- dV : Small element of volume.

The dot Product (.) :

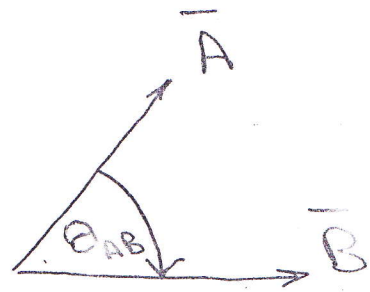
if we have 2 vectors $\bar{A}$ and $\bar{B}$:

$$\bar{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

$$\bar{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}$$

$$\bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_z B_z$$

$$= |A|\,|B| \cos\theta_{AB}$$



- Important notes:

(1) $\bar{A} \cdot \bar{A} = |A|\,|A| \cos(0) = |A|^2$

(2) $\bar{A} \cdot \bar{B} = \bar{B} \cdot \bar{A}$ (scalar not vector).

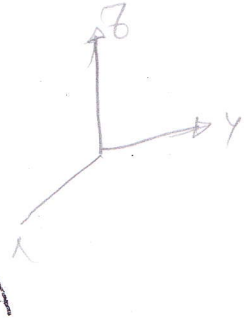
(3) $\hat{x} \cdot \hat{x} = |1|\,|1| \cos(0) = 1$

(4) $\hat{x} \cdot \hat{y} = |1|\,|1| \cos(90) = 0$

(5) any unit vector . any unit vector = $\cos($ الزاوية المحصورة $)$.

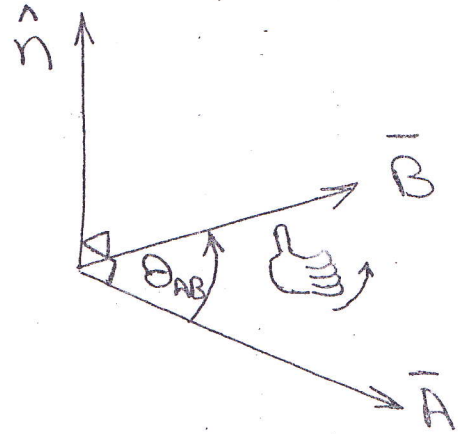
② The Cross Product (×):

$$\Rightarrow \bar{A} \times \bar{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$



$$= |A|\,|B| \sin\theta_{AB}\, \hat{n}$$

$\hat{n}$: هو متجه الوحدة العمودى على المستوى الذى يحوى $\bar{A}$ و $\bar{B}$ ويتحدد إتجاهه عن طريق قاعدة اليد اليمنى.



- Important notes:

(1) $\bar{A} \times \bar{A} = |A|\,|A| \sin(0) = 0$

(2) $\bar{A} \times \bar{B} = -(\bar{B} \times \bar{A})$

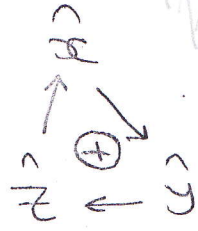
(3) $\hat{x} \times \hat{y} = \hat{z}$

(4) $\hat{y} \times \hat{z} = \hat{x}$

(5) $\hat{x} \times \hat{z} = -\hat{y}$

$\Rightarrow$

(6) $\hat{x} \times \hat{x} = 0$



ex.2 :

let $\vec{A} = 2\hat{x} - 3\hat{y}$ and $\vec{B} = 5\hat{x} + 4\hat{z}$

find $\vec{A} \times \vec{B}$ and $\vec{A} \cdot \vec{B}$?

Solution

$$\Rightarrow \vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & -3 & 0 \\ 5 & 0 & 4 \end{vmatrix}$$

$$= -12\hat{x} - 8\hat{y} + 15\hat{z}$$

$$\Rightarrow \vec{A} \cdot \vec{B} = 2*5 - 3*0 + 0*4 = 10$$

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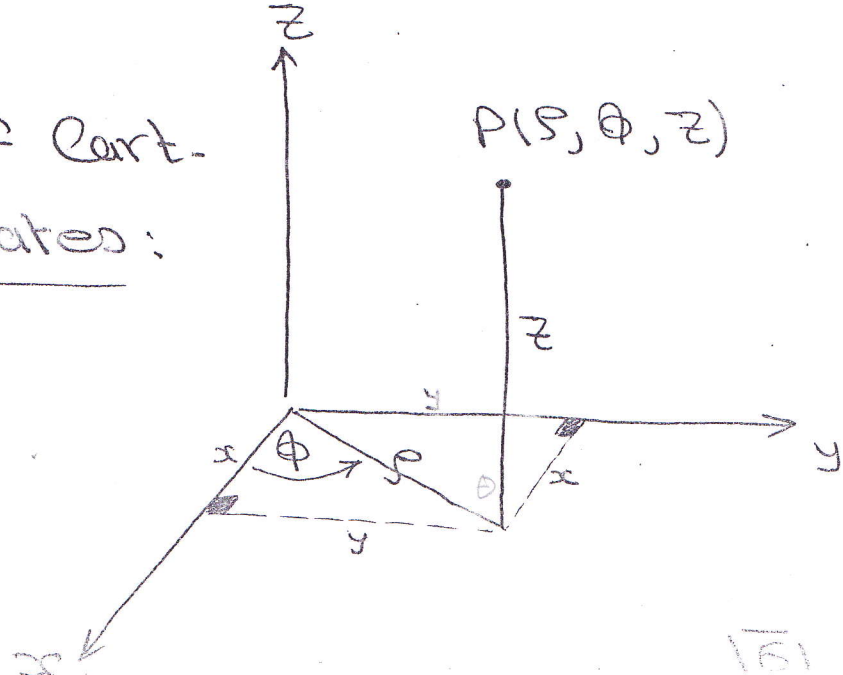
[2] Cylinderical Coordinate Sys :

(a) The relation bet^n Cart. and Cyl. Coordinates :



- Cart $\longrightarrow$ Cyl :

$$\rho = \sqrt{x^2 + y^2}$$

$$\Phi = \tan^{-1}\left(\frac{y}{x}\right)$$

• Cyl. ⟶ Cart. :

$x = \rho \cos\phi$

$y = \rho \sin\phi$

يلاحظ أن إحداثي ال z ثابت في كلا النظامين

⇛ Transformation table :

	$\hat{\rho}$	$\hat{\phi}$	$\hat{z}$
$\hat{x}$	$\cos\phi$	$-\sin\phi$	0
$\hat{y}$	$\sin\phi$	$\cos\phi$	0
$\hat{z}$	0	0	1

example : if $\bar{A} = 3\hat{x}$ (in Cart. Coo.)

$\therefore \bar{A} = 3[\cos\phi\,\hat{\rho} - \sin\phi\,\hat{\phi}]$ (in Cyl. Coo).

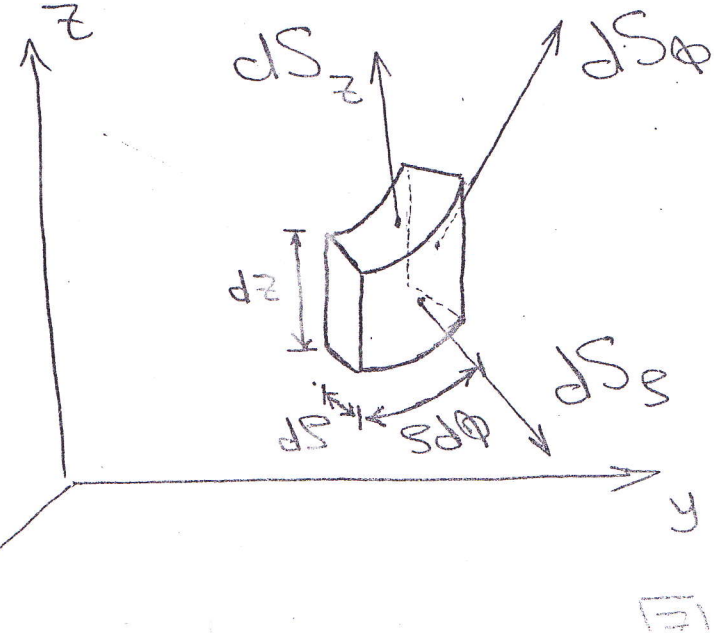
(b) Volume and surface area elements :

$dS_\rho = \rho\, d\phi\, dz$

$dS_\phi = d\rho\, dz$

$dS_z = \rho\, d\rho\, d\phi$

$dV = \rho\, d\rho\, d\phi\, dz$



ex. 3 : Transform $\bar{A}$ to Cyl. Coo.

$$\bar{A} = 3\hat{x} + 4\hat{y} - 2\hat{z}$$

{ Solution }

From transformation table :

$$\bar{A} = 3[\cos\phi\,\hat{\rho} - \sin\phi\,\hat{\phi}] + 4[\sin\phi\,\hat{\rho} + \cos\phi\,\hat{\phi}] - 2\hat{z}$$

$$\bar{A} = (3\cos\phi + 4\sin\phi)\hat{\rho} + (4\cos\phi - 3\sin\phi)\hat{\phi} - 2\hat{z}$$

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3) The Spherical Coordinate Sys :

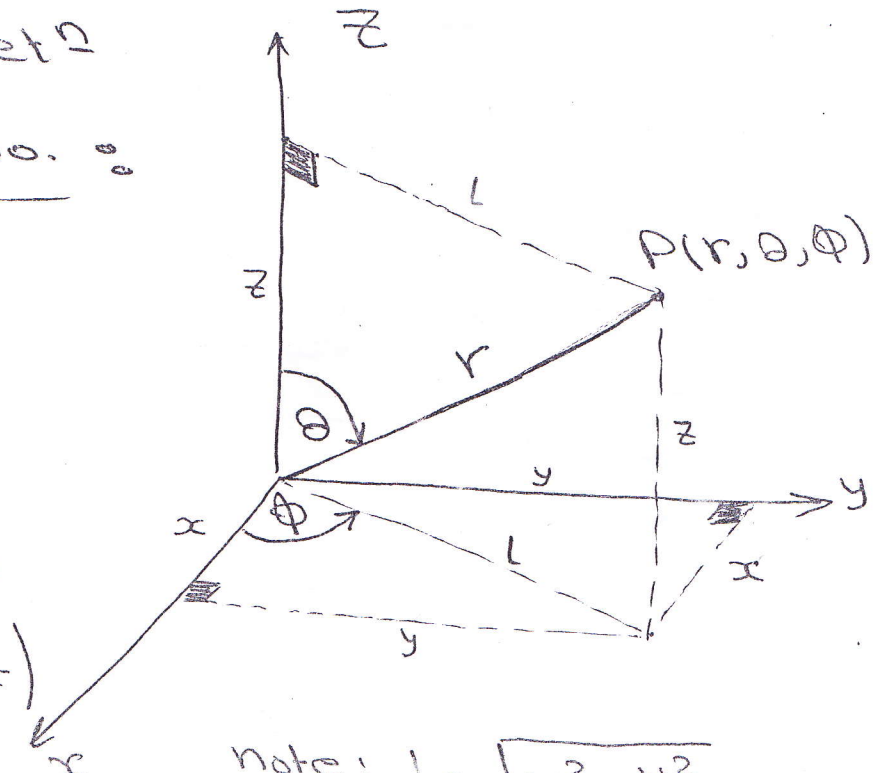
a) The relation bet^n Cart. and Sph. Coo. :

- Cart. ⟶ Sph.

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right) = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$



note: $l = \sqrt{x^2 + y^2}$

$r = \sqrt{l^2 + z^2}$

- Sph ⟶ Cart :

$$x = r \sin\theta \cos\phi$$

$$y = r \sin\theta \sin\phi$$

$$z = r \cos\theta$$

يلاحظ أن إحداثي ϕ ثابت في نظام الـ Sph هو نفسه في نظام الـ Cyl.

⇛ Transformation table :

	$\hat{r}$	$\hat{\theta}$	$\hat{\phi}$
$\hat{x}$	$\sin\theta \cos\phi$	$\cos\theta \cos\phi$	$-\sin\phi$
$\hat{y}$	$\sin\theta \sin\phi$	$\cos\theta \sin\phi$	$\cos\phi$
$\hat{z}$	$\cos\theta$	$-\sin\theta$	0

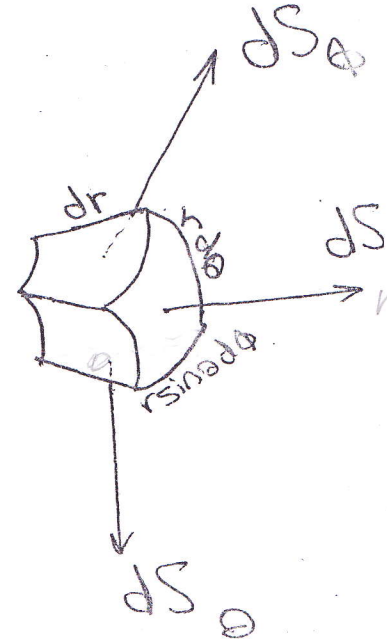
(b) Volume and surface area elements :

$$dS_r = r^2 \sin\theta \, d\theta \, d\phi$$

$$dS_\theta = r \sin\theta \, dr \, d\phi$$

$$dS_\phi = r \, dr \, d\theta$$

$$dV = r^2 \sin\theta \, dr \, d\theta \, d\phi .$$



Part (2): Important definations

⇒ The Coordinate sys parameters table:

Coo.	h_1	h_2	h_3	u_1	u_2	u_3
Cart.	1	1	1	x	y	z
Cyl.	1	ρ	1	ρ	ϕ	z
Sph	1	r	$r\sin\theta$	r	θ	ϕ

(1) The gradiant: (Scalar ⟶ Vector)

$$\nabla V \equiv \text{grad}(V)$$

(V ⟶ Scalar)

$$\nabla V = \frac{1}{h_1}\frac{\partial V}{\partial u_1}\hat{u}_1 + \frac{1}{h_2}\frac{\partial V}{\partial u_2}\hat{u}_2 + \frac{1}{h_3}\frac{\partial V}{\partial u_3}\hat{u}_3$$

(2) The divergence: (Vector ⟶ Scalar)

$$\nabla \cdot \bar{D} = \text{div}(\bar{D})$$

($\bar{D}$ ⟶ Vector)

$$\nabla \cdot \bar{D} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}(h_2 h_3 D_{u1}) + \frac{\partial}{\partial u_2}(h_1 h_3 D_{u2}) + \frac{\partial}{\partial u_3}(h_1 h_2 D_{u3})\right]$$

(3) The laplacian : (sCalar ⟶ sCalar)

$$\nabla^2 V = \nabla \cdot \nabla V \equiv \text{laplacian}(V)$$

$$= \text{div}(\text{grad } V)$$

↳ sCalar

$$\nabla^2 V = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2 h_3}{h_1}\frac{\partial V}{\partial u_1}\right) + \frac{\partial}{\partial u_2}\left(\frac{h_1 h_3}{h_2}\frac{\partial V}{\partial u_2}\right) + \frac{\partial}{\partial u_3}\left(\frac{h_1 h_2}{h_3}\frac{\partial V}{\partial u_3}\right)\right]$$

(4) The Curl : (vector ⟶ vector)

$$\nabla \times \bar{D} \equiv \text{Curl}(\bar{D})$$

↳ vector

$$\nabla \times \bar{D} = \frac{1}{h_1 h_2 h_3}\begin{vmatrix} h_1\hat{u}_1 & h_2\hat{u}_2 & h_3\hat{u}_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1 D_{u1} & h_2 D_{u2} & h_3 D_{u3} \end{vmatrix}$$

Ex. 4 :

Find ∇A if $A = 3r\theta$

Solution

∵ A is represented by Sph. Coo. :

$\therefore h_1 = 1, \; h_2 = r, \; h_3 = r\sin\theta$

$u_1 = r, \; u_2 = \theta, \; u_3 = \phi$

by substitution in the gradiant form :

$$\therefore \nabla V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}$$

$$\therefore \nabla A = \underbrace{\frac{\partial A}{\partial r}}_{\downarrow 3\theta}\hat{r} + \frac{1}{r}\underbrace{\frac{\partial A}{\partial \theta}}_{\downarrow 3r}\hat{\theta} + \frac{1}{r\sin\theta}\underbrace{\frac{\partial A}{\partial \phi}}_{\downarrow \text{zero}}\hat{\phi}$$

$$= (3\theta)\hat{r} + \frac{1}{r}(3r)\hat{\theta}$$

$$= 3\theta\,\hat{r} + 3\hat{\theta}$$

Sheet (1)

[1] a(1, 2, 3) b(2, 3, -5) c(-2, 4, 7)

(a) $\vec{A} = b - a = \hat{x} + \hat{y} - 8\hat{z}$

(b) $\vec{B} = c - a = -3\hat{x} + 2\hat{y} + 4\hat{z}$

(c) $|A| = \sqrt{1+1+64} = \sqrt{66}$

$|B| = \sqrt{9+4+16} = \sqrt{29}$

$$\hat{A} = \frac{\vec{A}}{|A|} = \frac{1}{\sqrt{66}}\hat{x} + \frac{1}{\sqrt{66}}\hat{y} - \frac{8}{\sqrt{66}}\hat{z}$$

$$\hat{B} = \frac{\vec{B}}{|B|} = -\frac{3}{\sqrt{29}}\hat{x} + \frac{2}{\sqrt{29}}\hat{y} + \frac{4}{\sqrt{29}}\hat{z}$$

(d) $\vec{A} \cdot \vec{B} = -3 + 2 - 32 = -33$

(e)

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 1 & -8 \\ -3 & 2 & 4 \end{vmatrix}$$

$$= 20\hat{x} + 20\hat{y} + 5\hat{z}$$

(b) Trans. table:

	$\hat{\rho}$	$\hat{\phi}$	$\hat{z}$
$\hat{x}$	$\cos\phi$	$-\sin\phi$	0
$\hat{y}$	$\sin\phi$	$\cos\phi$	0
$\hat{z}$	0	0	1

$$\therefore \bar{A} = \cos\phi\,\hat{\rho} - \sin\phi\,\hat{\phi} + \sin\phi\,\hat{\rho} + \cos\phi\,\hat{\phi} - 8\,\hat{z}$$

$$= [\cos\phi + \sin\phi]\hat{\rho} + [\cos\phi - \sin\phi]\hat{\phi} - 8\,\hat{z}$$

at Point $(3, \pi/3, 2)$

$$\therefore \phi = \pi/3 \text{ rad.}$$

$$\therefore \bar{A} = 1.366\,\hat{\rho} - 0.366\,\hat{\phi} - 8\,\hat{z}$$

(g) Trans. table:

	$\hat{r}$	$\hat{\theta}$	$\hat{\phi}$
$\hat{x}$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\hat{y}$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\hat{z}$	$\cos\theta$	$-\sin\theta$	0

$$\bar{B} = -3\hat{x} + 2\hat{y} + 4\hat{z}$$

$$= -3[\sin\theta\cos\phi\,\hat{r} + \cos\theta\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi}]$$

$$+ 2[\sin\theta\sin\phi\,\hat{r} + \cos\theta\sin\phi\,\hat{\theta} + \cos\phi\,\hat{\phi}]$$

$$+ 4[\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}]$$

$$\bar{B} = (-3\sin\theta\cos\phi + 2\sin\theta\sin\phi + 4\cos\theta)\hat{r}$$

$$+ (-3\cos\theta\cos\phi + 2\cos\theta\sin\phi - 4\sin\theta)\hat{\theta}$$

$$+ (3\sin\phi + 2\cos\phi)\hat{\phi}$$

at point $(3, \pi/4, \pi)$

$\therefore \theta = \pi/4 \qquad \phi = \pi.$

$$\therefore \bar{B} = 4.95\,\hat{r} - 0.707\,\hat{\theta} - 2\hat{\phi}$$

(h) let $\bar{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$

Prove $\nabla\cdot(\nabla\times\bar{A}) = 0$

$$\nabla\times\bar{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\therefore \nabla \times \bar{A} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\hat{x}$$

$$-\left[\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right]\hat{y} + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\hat{z}$$

$$\nabla \cdot (\nabla \times \bar{A}) = \frac{\partial}{\partial x}\left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]$$

$$-\frac{\partial}{\partial y}\left[\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right] + \frac{\partial}{\partial z}\left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]$$

$$\therefore \nabla \cdot (\nabla \times \bar{A}) = \frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_y}{\partial x \partial z} - \frac{\partial^2 A_z}{\partial y \partial x}$$

$$+ \frac{\partial^2 A_x}{\partial y \partial z} + \frac{\partial^2 A_y}{\partial z \partial x} - \frac{\partial^2 A_x}{\partial z \partial y}$$

$$= 0 \quad \#$$

(I)

$$-\nabla V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}$$

$$-\nabla V = 3\sin\phi\,\hat{r} + 0\,\hat{\theta} + \frac{1}{r\sin\theta}\,3r\cos\phi\,\hat{\phi}$$

$$= 3\sin\phi\,\hat{r} + 3\,\frac{\cos\phi}{\sin\theta}\,\hat{\phi}$$

• $\nabla \cdot \nabla V = \nabla^2 V$

$$\nabla^2 V = \frac{1}{r^2 \sin\theta}\left[\frac{\partial}{\partial r}\left(r^2 \sin\theta \frac{\partial V}{\partial r}\right)\right.$$

$$\left. + \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{\partial}{\partial \phi}\left(\frac{1}{\sin\theta}\frac{\partial V}{\partial \phi}\right)\right]$$

$$\nabla^2 V = \frac{1}{r^2 \sin\theta}\left[\frac{\partial}{\partial r}(3r^2 \sin\theta \sin\phi)\right.$$

$$\left. + \frac{\partial}{\partial \phi}\left(\frac{3r\cos\phi}{\sin\theta}\right)\right]$$

$$= \frac{1}{r^2 \sin\theta}\left[6r\sin\theta\sin\phi - \frac{3r\sin\phi}{\sin\theta}\right]$$

$$\therefore \nabla^2 V = \frac{6\sin\phi}{r} - \frac{3\sin\phi}{r\sin^2\theta}$$

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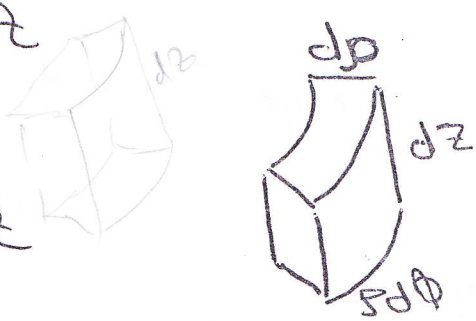
[2] Cyl. $dv = s\,ds\,d\phi\,dz$

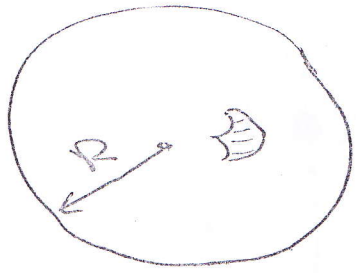
$$\therefore V = \int_0^1 \int_0^{\pi/3} \int_1^2 s\,ds\,d\phi\,dz$$

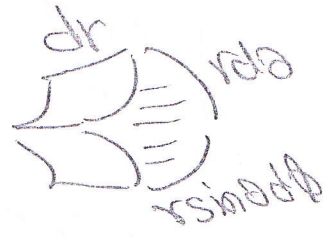


$$= \frac{s^2}{2}\Big|_1^2 * \frac{\pi}{3} * 1$$

$$= \frac{\pi}{6}[4-1] = \frac{\pi}{2}\ m^3$$

[3]





$\therefore dS_r = r^2 \sin\theta \, d\theta \, d\phi .$

$$\therefore S = \int_0^{2\pi} \int_0^{\pi} r^2 \sin\theta \, d\theta \, d\phi$$

for the surface area $r = R$.

$$\therefore S = \int_0^{2\pi} \int_0^{\pi} R^2 \sin\theta \, d\theta \, d\phi$$

$$= R^2 * \underbrace{\cos\theta}_{2} \Big|_{\pi}^{0} * 2\pi$$

$$= 4\pi R^2 .$$

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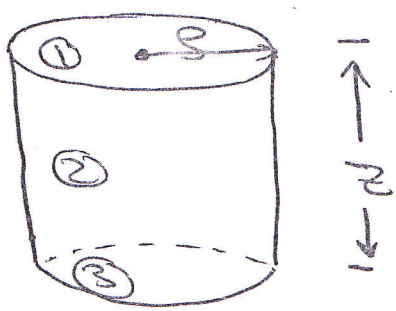
[4]



$S = 3$

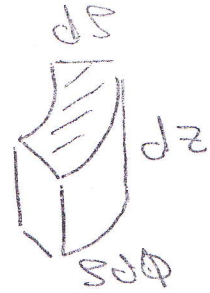
$z = 3$

$\iint \rho \, d\rho \, d\phi$

We have 3 surfaces : ①, ②, ③

Sur ① = Sur ③

Sur ① :

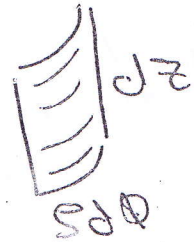


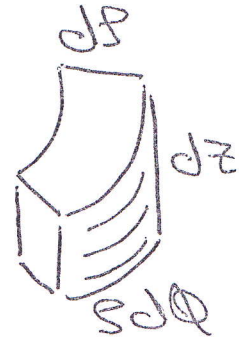
$$\therefore dS_z = \rho \, d\rho \, d\phi$$

$$\therefore S_1 = \int_0^{2\pi} \int_0^3 \rho \, d\rho \, d\phi$$

$$= 2\pi * \frac{\rho^2}{2} \Big|_0^3 = 9\pi$$

Sur ② :





$$dS_\rho = \rho \, d\phi \, dz$$

on sur ② : $\rho = 3$

$$\therefore dS_\rho = 3 \, d\phi \, dz$$

$$S_\rho = \int_0^3 \int_0^{2\pi} 3 \, d\phi \, dz$$

$$S_2 = 3 * 2\pi * 3 = 18\pi$$

$$\therefore S = S_1 + S_2 + S_3 = 2S_1 + S_2$$

$$= 2 * 9\pi + 18\pi = 36\pi \ m^2$$